TRINITY COLLEGE

Mathematics Specialist Units 3,4
Test 2 2018

Section 1 Calculator Free

Vectors

STUDENT'S NAME _____

DATE: Thursday 29 March **TIME:** 20 minutes **MARKS**: 22

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

The points A, B and C have position vectors $\mathbf{a} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ and $\mathbf{c} = \begin{pmatrix} 7 \\ -3 \\ 10 \end{pmatrix}$ respectively. Show that angle ABC is a right angle.

2. (5 marks)

(a) A sphere has Cartesian equation $x^2 + y^2 + z^2 - 2x + 4y - 6z = 11$. Determine the coordinates of the centre and the radius of the sphere. (4 marks)

(b) Write the vector equation of the sphere from part (a) (1 mark)

3. (8 marks)

The position vectors of the points A and B are $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 5 \end{pmatrix}$ respectively.

(a) Determine the vector equation of the line joining A and B. (2 marks)

(b) Determine the cartesian equation of the line joining A and B. (3 marks)

(c) Determine the cosine of the angle the line AB makes with the x-axis. (3 marks)

4. (5 marks)

Determine a unit vector perpendicular to the plane that contains the points with position vectors $A\begin{pmatrix}1\\2\\1\end{pmatrix}$, $B\begin{pmatrix}-2\\1\\3\end{pmatrix}$ and $C\begin{pmatrix}0\\2\\4\end{pmatrix}$.

TRINITY COLLEGE

Mathematics Specialist Units 3,4
Test 2 2018

Section 2 Calculator Assumed

Vectors

STUDENT'S NAME _____

DATE: Thursday 29 March **TIME:** 30 minutes **MARKS**: 32

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (5 marks)

Two particles A and B have initial position vectors $-4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ respectively and their respective constant velocity vectors are $2\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$. (Units are metres and seconds)

(a) Determine the position vector of where the paths of the particles cross. (3 marks)

(b) Do the particles meet? Explain. (2 marks)

6. (5 marks)

The position vector of a particle is given by $\mathbf{r} = \begin{pmatrix} 2 + 4\sin 2t \\ 4\cos 2t - 3 \end{pmatrix}$, where t is time in seconds.

(a) State the initial position vector of the particle. (1 mark)

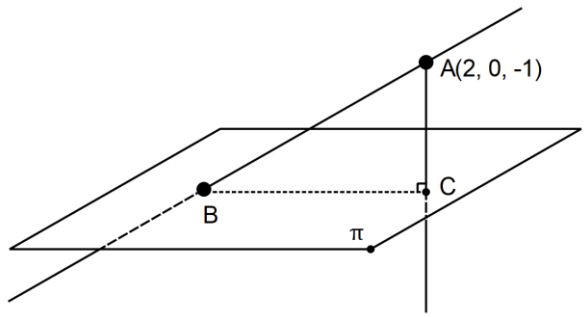
(b) Determine the Cartesian equation of the path of the particle. (4 marks)

7. (5 marks)

Determine the minimum distance between the point with position vector $\begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix}$ and the plane $2x - 4y + z = 1$.

8. (8 marks)

The point A has coordinates (2, 0, –1) and the plane Π has the equation $x + 2y - 2z = 8$. The line through A parallel to the line $\frac{x}{-2} = y = \frac{z+1}{2}$ meets Π at B and the perpendicular from A to Π meets Π at the point C as shown in the diagram.



(a) Determine the equations of the two lines AB and AC. (4 marks)

(b) Determine the coordinates of the points B and C. (4 marks)

9. (9 marks)

A plane Π contains two lines $\mathbf{r} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{j} + 7\mathbf{k})$ and $\mathbf{r} = \mathbf{i} - \mathbf{j} + 2\mathbf{k} + \mu(-5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k})$.

(a) Write down a vector equation of the plane Π in the form $\mathbf{r} = \mathbf{a} + \alpha\mathbf{b} + \beta\mathbf{c}$. (1 mark)

(b) The point $-9\mathbf{i} + 2\mathbf{j} + n\mathbf{k}$ lies in the plane Π. Determine the value of the constant n. (3 marks)

(c) The vector $m\begin{pmatrix} 5 \\ 7 \\ b \end{pmatrix}$ is normal to the plane. Determine the values of the constants m and b. (3 marks)

(d) State the equation of the plane Π in the form $\mathbf{r} \cdot \mathbf{n} = c$. (2 marks)